Direct Proofs

~~For all $a \in \mathbb{R}$, $a \in a$~~

There exists $a \in \mathbb{R}$ for which

$a + x = x$ for all $x \in \mathbb{R}$

~~read x;~~

~~$\forall a \in \mathbb{R}$~~

$\exists a \in \mathbb{R} \left(\forall x \in \mathbb{R} \quad a + x = x \right)$

Negation

$\forall a \in \mathbb{R} \quad \exists x \in \mathbb{R} \quad a + x \neq x$

For all $a \in \mathbb{R}$ for which $a + x \neq x$ for some $x \in \mathbb{R}$

$X_i = \{ x \in \mathbb{R} \quad 2i \leq x \leq 2i+1 \}$

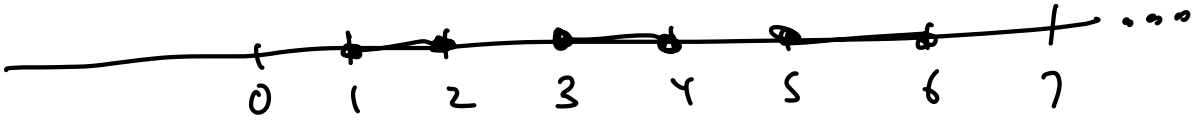
$i \in \mathbb{N} \qquad i = 1, 2, 3, \ldots$

$\bigcup_i X_i$

$X_1 : 2 \leq x \leq 3$

$X_2 : 4 \leq x \leq 5$

$X_3 : 6 \leq x \leq 7$



If P, then Q

Proposition: If P is true, then Q is true.

We are asserting the truth of the implication $P \implies Q$. If P is false, then the implication is automatically true. So from a practical point of view, we must show that whenver P is true, Q is also true.

Proof: Suppose P.

(bunch of stuff)

Therefore Q.

Simple example

Proposition: The sum of two odd integers is even.

x is odd $\iff$ $x = 2q+1$ where $q \in \mathbb{Z}$. (division w/ remainder)

x divided by 2 has rem $\nearrow$ 0 x even
$\searrow$ 1 x odd

Let x and y be odd integers.

Then there are integers q_1 and q_2 such that

$$x = 2q_1 + 1$$
$$y = 2q_2 + 1$$

by the division algorithm.

$$x+y = (2q_1+1)+(2q_2+1) = 2q_1+2q_2+2$$
$$= 2(q_1+q_2+1)$$

Therefore $x+y = 2s$ where $s = q_1+q_2+1 \in \mathbb{Z}$ so $x+y$ is even.

Proposition: The sum of two odd integers is even.

Proof:

Suppose a and b are odd integers.

Therefore $a + b$ is even.

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Suppose a and b are odd integers.

Then there are integers k and u so that $a = 2k + 1$ and $b = 2u + 1$.

Therefore $a + b$ is even.

Proposition: The sum of two odd integers is even.

Proof:

Suppose a and b are odd integers.

Then there are integers k and u so that $a = 2k + 1$ and $b = 2u + 1$.

Therefore

$$a + b = (2k + 1) + (2u + 1) = 2(k + u) + 2 = 2(k + u + 1)$$

Therefore $a + b$ is even.

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Proof:

Suppose a and b are odd integers.

Then there are integers k and u so that $a = 2k + 1$ and $b = 2u + 1$.

Therefore $a + b = (2k + 1) + (2u + 1) = 2(k + u) + 2 = 2(k + u + 1)$

The definition of even integer says that an integer x is even if there is an integer y so that $x = 2y$. We found that $a + b = 2(k + u + 1)$ where $k + u + 1$ is an integer.

Therefore $a + b$ is even.